SEMESTRAL EXAMINATION
B. MATH II YEAR
ANALYSIS IV
II SEMESTER, 2010-2011

The 7 questions carry a total of 110 marks. Answer as many questions as you can. The maximum you can score is 100. Time limit is 3 hours.

1. Let $f : [0,1] \to [0,1]$ be continuous and bijective. Let $g : [0,1] \to [0,1]$ be a bounded measurable function with $\int_0^1 g(x)[f(x)]^n dx = 0$ for $n = 0, 1, 2, \ldots$. Show that $\int_\alpha^\beta g(x)dx = 0$ whenever $0 \leq \alpha \leq \beta \leq 1$. [20]

2. Let $f(x,y) = g(x)y$, $-1 \leq x \leq 1, -1 \leq y \leq 1$ where g is a continuous function with values in $[-1,1]$. Using Picard's Theorem describe a procedure for solving the differential equation $y' = f(x,y)$ with the initial condition $y(0) = 1/2$. Show that the solution exists in $[-1/4, 1/4]$. [20]

3. Which of the following sequences in $C[0,1]$ are equicontinuous? Justify.

a) $f_n(x) = (e^x - 5)^n$

b) $f_n(x) = (e^{-x} - 5)^n$

c) $f_n(x) = e^{-nx}$ [15]

4. Consider the space $L^1[-\pi,\pi]$. Show that there is no identity element for the operation of convolution on this space. [15]

5. Find a periodic differentiable function $\phi : \mathbb{R} \to \mathbb{R}$ with period 2π such that $\phi'(t) + 2\phi(t - \pi) = \sin t$ for all $t \in \mathbb{R}$. [15]

Hint: guess what ϕ should be using Fourier coefficients.

6. Let f be of bounded variation, periodic with period 2π and right continuous on $[-\pi,\pi]$. If the Fourier series of f converges to f at each point show that f is necessarily uniformly continuous on $\mathbb{R}$. [10]

7. Prove that the Fourier series of $(\pi - x)e^{|x|}$ $(-\pi \leq x \leq \pi)$ converges at each point of $(-\pi/2, \pi/2)$. [15]